Weakest link clustering and convex geometry II

Boris Mirkin (National Research University Higher School of Economics, Russia)

DATA ANALYSIS AND MODELING LAB

Palacky University, Olomouc, Czech Republic

european social fund in the czech republic

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Boris Mirkin lecture 2d part

Set-to-Subset Mappings

and

Set Systems

related to

Monotone Linkage Functions

(Muchnik and Co 1988-1990)

(Mirkin and Muchnik, 2002)

Hereditary and Isotone Mappings from Monotone Linkage I

- **$\varphi_u(S)=\{ k: d(k,S)\leq u\}$** **(assume not empty!)**

- "exterior" elements

- **$\psi_u(S)=\{ k: d(k,S)>u\} = S - \varphi_u(S)$**

- "interior" elements

$$\varphi_u(S)\cup\psi_u(S)=S$$

Hereditary and Isotone Mappings from Monotone Linkage II

- $\varphi_u(S)=\{ k: d(k,S)\leq u\}$ (assume not empty!)
- $\psi_u(S)=\{ k: d(k,S)>u\} =S - \varphi_u(S)$

Isotone $\psi_u(S)$:

$$\psi_u(S) \cup \psi_u(T) \subseteq \varphi_u(S\cup T)$$

Hereditary $\varphi_u(S)$:

$$\varphi_u(S\cup T) \subseteq \varphi_u(S)\cup \varphi_u(T)$$

$$\varphi_u(S\cup T) \cap S \subseteq \varphi_u(S) \quad \text{(same)}$$

Any isotone mapping and any hereditary mapping can be generated in this way

Interior set system

Given a mapping **φ,** **φ(S)≠∅** if **S≠∅,**

Interior set system $C(\varphi) \subseteq 2^I$:

1. $I \in C(\varphi)$

2. If $S \in C(\varphi)$, then $T \in C(\varphi)$ if $S - \varphi(S) \subseteq T \subseteq S$

Any interior set system can be generated as the set of all patterns for a monotone pair (d(k,S), u) ⇒φ with all possible monotone π(k,S) (in fact, only those set-characteristic, p_A)

Convex Geometry (Edelman, Jamison 1985)

Convex geometry $L \subseteq \mathbf{2^I}$:

- **L1.** $\varnothing$**, I**$\in$ **L**
- **L2.** **A, B** $\in$ **L** $\Rightarrow$ **A**$\cap$**B** $\in$ **L**
- **L3.** **A**$\in$ **L** $\Rightarrow$ **A+i** $\in$ **L for some i** $\in$ **I-A**

An abstract lattice characteristic (Edelman…)

Another characteristic: **Anti-matroid** (everywhere)

I like Generator: series/orders of all elements of I

Filter

$$F(p) = \{\{i_N\}, \{i_{N-1}, i_N\}, \{i_{N-2}, i_{N-1}, i_N\}, \ldots, \{i_1, \ldots, i_{N-1}, i_N\}\}$$

for $p = i_1 i_2 \ldots i_{N-1} i_N$

Convex Geometry Generator

Consider a series/order of all elements of I

$p=i_1 i_2 \dots i_{N-1} i_N$

Its filter (all ending fragments)

$F(p) = \{\{i_N\}, \{i_{N-1}, i_N\}, \{i_{N-2}, i_{N-1}, i_N\}, \dots, \{i_1, \dots, i_{N-1}, i_N\}\}$

Consider **series p1, p2, pm and set of all intersections**

$$L(p1,\dots,pm)=\bigcap_{k=1\dots m} F(pk)$$

This is a convex geometry; any convex geometry can be generated in this way

Convex geometry and interior set system

Convex geometry = Interior set system

Any convex geometry can be generated from constrained tightness functions

Conclusion: Concepts introduced

- Monotone linkage and tightness functions
- Quasi-convex set functions
- Shelled core clusters greedy-wise found
- Monotone linkage to generate hereditary mappings
- Hereditary mapping to generate convex geometries as interior set systems

Conclusion: Further work

- Mathematics
  - Base linkage system
  - Iterative mapping
  - Duality
  - Optimization and game theory
  - Choice mappings
- Applications
  - Organization systems
  - Network structure
  - Context table analysis