Modeling and learning preferences over conjunctive concepts

Sergei Obiedkov
(National Research University Higher School of Economics, Russia)

DAMOL
DATA ANALYSIS AND MODELING LAB
Palacky University, Olomouc, Czech Republic

european social fund in the czech republic
EUROPEAN UNION
MINISTRY OF EDUCATION, YOUTH AND SPORTS
OP Education for Competitiveness
INVESTMENTS IN EDUCATION DEVELOPMENT

- John chooses strawberries over apples.
- John chooses raspberries over pears.

Can we generalize?

- John likes red berries more than tree fruit.

- John chooses strawberries over apples.
- John chooses raspberries over pears.

Can we generalize?

- John likes red berries more than tree fruit.

1. What exactly do we mean by *likes more than*?
2. How do we step from preferences over objects to preferences over concepts?
    - What is the right level of generalization (*berries* vs. *red berries*)?

Two ways to specify concepts:

intensionally, by describing what it takes to be an instance of the concept;

extensionally, by enumerating instances of the concept.

Cf. a logical formula and the set of its models.

Two ways to specify concepts:

intensionally, by describing what it takes to be an instance of the concept;

extensionally, by enumerating instances of the concept.

Cf. a logical formula and the set of its models.

Thus,

- preferences over concepts are
- preferences over descriptions are
- preferences over sets of objects.

"Lifting" preferences to propositions/attribute sets

In preference logics (van Benthem et al. 2009):

$$\phi \text{ is preferred to } \psi$$
$$\Updownarrow$$
$$\text{models of } \phi \text{ are preferred to models of } \psi$$

Our approach is similar, but:

- We consider only non-strict preferences (so far).

"Lifting" preferences to propositions/attribute sets

In preference logics (van Benthem et al. 2009):

$$\phi \text{ is preferred to } \psi$$
$$\Updownarrow$$
$$\text{models of } \phi \text{ are preferred to models of } \psi$$

Our approach is similar, but:

- We consider only non-strict preferences (so far).
- We allow only conjunctive propositions for ϕ and ψ, treating them as attribute subsets.

The goal is to describe a complete set of preferences for a given interpretation.

Formal Concept Analysis
(Ganter and Wille 1999)

Formal context $\mathbb{K} = (G, M, I)$

- a set of objects G
- a set of attributes M
- objects are described with attributes: the binary relation $I \subseteq G \times M$

Formal Concept Analysis
(Ganter and Wille 1999)

Formal context $\mathbb{K} = (G, M, I)$

- a set of objects G
- a set of attributes M
- objects are described with attributes: the binary relation $I \subseteq G \times M$

Derivation operators

For $A \subseteq G$ and $B \subseteq M$:

- $A' = \{m \in M \mid \forall g \in A : gIm\}$
- $B' = \{g \in G \mid \forall m \in B : gIm\}$

Formal concept (A, B)

$A \subseteq G, B \subseteq M \qquad A' = B, B' = A$

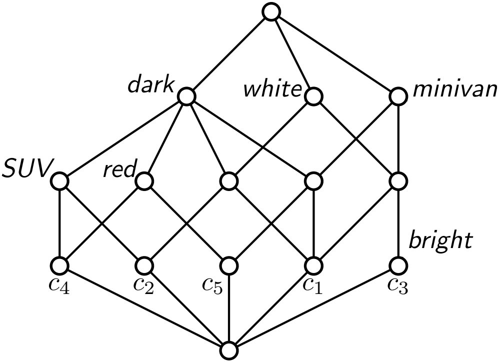
Example

adapted from (Brafman and Domshlak 2009)

Context

	minivan	SUV	red exterior	white exterior	bright interior	dark interior
c_1	×			×		×
c_2		×		×		×
c_3	×			×	×	
c_4		×	×			×
c_5	×		×			×

Concept lattice



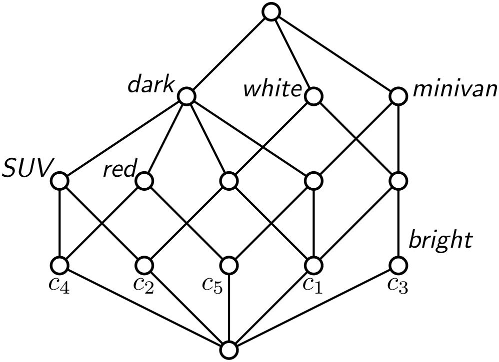
Example

adapted from (Brafman and Domshlak 2009)

Context

	minivan	SUV	red exterior	white exterior	bright interior	dark interior
c_1	×			×		×
c_2		×		×		×
c_3	×			×	×	
c_4		×	×			×
c_5	×		×			×

Concept lattice



Implication $A \to B$ holds in the context (G, M, I) if $A' \subseteq B'$

$\text{bright} \to \text{minivan}, \text{white}$ $\quad$ $\text{SUV} \to \text{dark}$ $\quad$ $\text{red} \to \text{dark}$

$\text{minivan}, \text{SUV} \to \bot$ $\quad$ $\text{red}, \text{white} \to \bot$ $\quad$ $\text{bright}, \text{dark} \to \bot$

Preference context $\mathbb{P} = (G, M, I, \leq)$

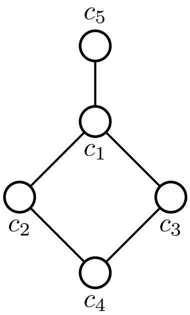
adapted from (Brafman and Domshlak 2009)

Cars (G, M, I)

	minivan	SUV	red exterior	white exterior	bright interior	dark interior
c_1	×			×		×
c_2		×		×		×
c_3	×			×	×	
c_4		×	×			×
c_5	×		×			×

Preferences

$\leq$ is a preorder on G



Preference context $\mathbb{P} = (G, M, I, \leq)$

adapted from (Brafman and Domshlak 2009)

Cars (G, M, I)

	minivan	SUV	red exterior	white exterior	bright interior	dark interior
c_1	×			×		×
c_2		×		×		×
c_3	×			×	×	
c_4		×	×			×
c_5	×		×			×

Preferences $(G, G, \leq)$

$\leq$ is a preorder on G

$$\{c_2, c_3\}^{\leq} = \{c_1, c_5\}$$
$$\{c_2, c_3\}^{\geq} = \{c_4\}$$

	c_1	c_2	c_3	c_4	c_5
c_1	×				×
c_2	×	×			×
c_3	×		×		×
c_4	×	×	×	×	×
c_5					×

Preference context $\mathbb{P} = (G, M, I, \leq)$

adapted from (Brafman and Domshlak 2009)

Cars (G, M, I)

	minivan	SUV	red exterior	white exterior	bright interior	dark interior
c_1	×			×		×
c_2		×		×		×
c_3	×			×	×	
c_4		×	×			×
c_5	×		×			×

Preferences $(G, G, \leq)$

$\leq$ is a preorder on G

$\{c_2, c_3\}^{\leq} = \{c_1, c_5\}$
$\{c_2, c_3\}^{\geq} = \{c_4\}$

	c_1	c_2	c_3	c_4	c_5
c_1	×				×
c_2	×	×			×
c_3	×		×		×
c_4	×	×	×	×	×
c_5					×

Semantics based on preference contexts

Valid preferences ($\mathbb{P} \models \pi$), semantic consequence ($\Pi \models \pi$), sound ($\mathbb{P} \models \Pi$) and complete sets of preferences. . .

Outline

- We define three types of preferences and, for each of them, consider the following

Problem

Input: *A preference context* $\mathbb{P}$*;*
Output: *A sound and complete set of preferences for* $\mathbb{P}$*.*

- We represent preferences by implications and cast the above problem as a problem of computing (some) implications of a (certain) formal context.
- The latter is a hard problem, but practically efficient algorithms do exist.

Types of preferences

In preference logics:

$$\phi \text{ is preferred to } \psi$$
$$\Updownarrow$$
$$\text{models of } \phi \text{ are preferred to models of } \psi$$

Types of preferences

In preference logics:

$$\phi \text{ is preferred to } \psi$$
$$\Updownarrow$$
$$\text{models of } \phi \text{ are preferred to models of } \psi$$

$\forall\forall$-preferences: every model of ϕ is preferred to every model of ψ

$\forall\exists$-preferences: for every model of ψ, there is a better model of ϕ

Types of preferences

In preference logics:

ϕ is preferred to ψ
$\Updownarrow$
models of ϕ are preferred to models of ψ

$\forall\forall$-preferences: every model of ϕ is preferred to every model of ψ

$\forall\exists$-preferences: for every model of ψ, there is a better model of ϕ

For $A \subseteq M$ and $B \subseteq M$:

A is preferred to B
$\Updownarrow$
A' is preferred to B'

Universal preferences over object sets

A set $Y \subseteq G$ is preferred to a set $X \subseteq G$ if

$$\forall x \in X \forall y \in Y (x \leq y)$$

Universal preferences over object sets

A set $Y \subseteq G$ is preferred to a set $X \subseteq G$ if

$$\forall x \in X \forall y \in Y (x \leq y)$$

or

$$Y \subseteq X^{\leq}.$$

- If Y is preferred to X, then the subsets of Y are preferred to the subsets of X.
- X and Y are maximal with respect to $Y \subseteq X^{\leq}$ if an only if (X, Y) is a formal concept of $(G, G, \leq)$.

The concepts of $(G, G, \leq)$ provide a complete representation of universal preferences over object sets.

Universal preferences over attribute sets

B is preferred to A $\iff$ B' is preferred to A'

Universal preferences over object sets

A set $Y \subseteq G$ is preferred to a set $X \subseteq G$ if

$$\forall x \in X \forall y \in Y (x \leq y).$$

Universal preferences over attribute sets

$\mathbb{P} \models A \preccurlyeq_{\forall} B$

A set $B \subseteq M$ is preferred to a set $A \subseteq M$ if

$$\forall x \in A' \forall y \in B' (x \leq y)$$

or

$$B' \subseteq A'^{\leq}.$$

Existential preferences

Existential preferences over object sets

A set $Y \subseteq G$ is preferred to a set $X \subseteq G$ if

$$\forall x \in X \exists y \in Y (x \leq y).$$

Existential preferences over attribute sets

$\mathbb{P} \models A \preccurlyeq_{\exists} B$

A set $B \subseteq M$ is preferred to a set $A \subseteq M$ if

$$\forall x \in A' \exists y \in B' (x \leq y)$$

Existential preferences

Existential preferences over object sets

A set $Y \subseteq G$ is preferred to a set $X \subseteq G$ if

$$\forall x \in X \exists y \in Y (x \leq y).$$

Existential preferences over attribute sets

$\mathbb{P} \models A \preccurlyeq_{\exists} B$

A set $B \subseteq M$ is preferred to a set $A \subseteq M$ if

$$\forall x \in A' \exists y \in B' (x \leq y)$$

or

$$A' \subseteq \bigcup_{g \in B'} g^{\geq}.$$

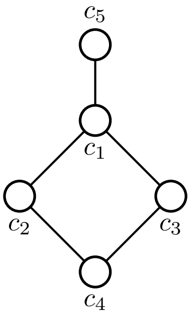
Existential preferences are generalized implications

For a preference context $\mathbb{P} = (G, M, I, \leq)$

1. If $(G, M, I) \models A \to B$, then $\mathbb{P} \models A \preccurlyeq_{\exists} B$.
2. If $\leq$ is the identity relation and $\mathbb{P} \models A \preccurlyeq_{\exists} B$, then $(G, M, I) \models A \to B$.

Example

	minivan	SUV	red exterior	white exterior	bright interior	dark interior
c_1	×			×		×
c_2		×		×		×
c_3	×			×	×	
c_4		×	×			×
c_5	×		×			×



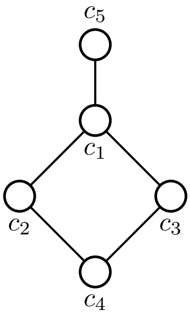
I prefer red minivans to white minivans.

$\text{minivan}, \text{white} \preccurlyeq_{\forall} \text{minivan}, \text{red}$

$\text{minivan}, \text{white} \preccurlyeq_{\exists} \text{minivan}, \text{red}$

Example

	minivan	SUV	red exterior	white exterior	bright interior	dark interior
c_1	×			×		×
c_2		×		×		×
c_3	×			×	×	
c_4		×	×			×
c_5	×		×			×



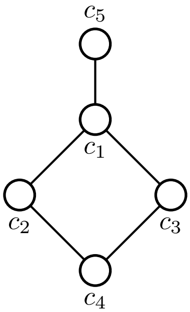
In white cars, I prefer a dark interior.

white, bright $\not\preccurlyeq_\forall$ white, dark

white, bright $\preccurlyeq_\exists$ white, dark

Example

	minivan	SUV	red exterior	white exterior	bright interior	dark interior
c_1	×			×		×
c_2		×		×		×
c_3	×			×	×	
c_4		×	×			×
c_5	×		×			×



In red cars, I prefer a bright interior.

red, dark $\preccurlyeq_\forall$ red, bright

red, dark $\not\preccurlyeq_\exists$ red, bright

Inference

Universal preferences

$$\frac{X \preccurlyeq_{\forall} Y}{X \cup U \preccurlyeq_{\forall} Y \cup V}$$

Existential preferences

$$\frac{}{X \preccurlyeq_{\exists} X} \qquad \frac{X \preccurlyeq_{\exists} Y \cup U}{X \cup V \preccurlyeq_{\exists} Y} \qquad \frac{X \preccurlyeq_{\exists} Y, \quad Y \preccurlyeq_{\exists} Z}{X \preccurlyeq_{\exists} Z}$$

Ceteris paribus preferences

Universal and existential preferences are global: propositions are compared w.r.t. all their models.

Ceteris paribus preferences

Universal and existential preferences are global: propositions are compared w.r.t. all their models.

Ceteris paribus preferences

- Preferences under normal conditions:

  John prefers red wine to white wine (but not when having fish).

Ceteris paribus preferences

Universal and existential preferences are global: propositions are compared w.r.t. all their models.

Ceteris paribus preferences

- Preferences under normal conditions:

  John prefers red wine to white wine (but not when having fish).

- Preferences assuming "everything else being equal":

  John prefers an academic job to a job in industry (other things being equal).

Ceteris paribus preferences

Universal and existential preferences are global: propositions are compared w.r.t. all their models.

Ceteris paribus preferences

- Preferences under normal conditions:

  John prefers red wine to white wine (but not when having fish).

- Preferences assuming "everything else being equal":

  John prefers an academic job to a job in industry (other things being equal).

- . . . or more specifically:

  John prefers an academic job to a job in industry (given the salary is the same).

Ceteris paribus preferences

Ceteris paribus preferences in preference logics

Y is preferred to X ceteris paribus with respect to a set Γ of propositions if

$$\forall x \in X \forall y \in Y (\forall \varphi \in \Gamma (x \models \varphi \iff y \models \varphi) \rightarrow x \leq y).$$

Ceteris paribus preferences

Ceteris paribus preferences in preference logics

Y is preferred to X ceteris paribus with respect to a set Γ of propositions if

$$\forall x \in X \forall y \in Y (\forall \varphi \in \Gamma (x \models \varphi \iff y \models \varphi) \to x \leq y).$$

Ceteris paribus preferences $\quad \mathbb{P} \models A \preccurlyeq_C B$

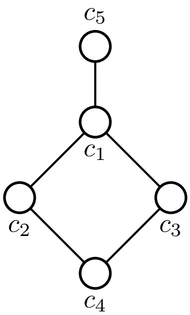
$B \subseteq M$ is preferred ceteris paribus to $A \subseteq M$ with respect to $C \subseteq M$ in $\mathbb{P} = (G, M, I, \leq)$ if

$$\forall g \in A' \forall h \in B' (\{g\}' \cap C = \{h\}' \cap C \to g \leq h).$$

This is a relaxation of universal preferences.

Example

	minivan	SUV	red exterior	white exterior	bright interior	dark interior
c_1	×			×		×
c_2		×		×		×
c_3	×			×	×	
c_4		×	×			×
c_5	×		×			×

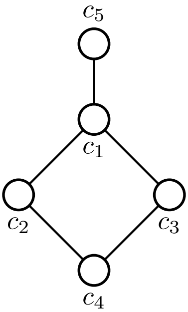


I prefer minivans to SUVs

SUV $\not\preceq_{\forall}$ minivan

Example

	minivan	SUV	red exterior	white exterior	bright interior	dark interior
c_1	×			×		×
c_2		×		×		×
c_3	×			×	×	
c_4		×	×			×
c_5	×		×			×



I prefer minivans to SUVs — SUV $\not\preccurlyeq_{\forall}$ minivan

… with the same interior color. — SUV $\preccurlyeq_{\{\text{bright,dark}\}}$ minivan

Ceteris paribus preferences as implications

Ceteris paribus translation of $\mathbb{P}$

$$\mathbb{K}^{\mathbb{P}}_{\sim} = (G \times G, (M \times \{1,2,3\}) \cup \{\leq\}, I_{\sim})$$

$$\begin{aligned}
(g_1, g_2) I_{\sim} (m, 1) &\iff g_1 I m, \\
(g_1, g_2) I_{\sim} (m, 2) &\iff g_2 I m, \\
(g_1, g_2) I_{\sim} (m, 3) &\iff \{g_1\}' \cap \{m\} = \{g_2\}' \cap \{m\}, \\
(g_1, g_2) I_{\sim} \leq &\iff g_1 \leq g_2.
\end{aligned}$$

Example

	m1	s1	r1	...	m2	s2	r2	...	m3	s3	r3	...	$\leq$
...													
c_1, c_4	×					×	×						
c_1, c_5	×				×		×		×	×			×
...													

Ceteris paribus preferences as implications

Translation of ceteris paribus preferences

A ceteris paribus preference $A \preccurlyeq_C B$ is valid in a preference context $\mathbb{P} = (G, M, I, \leq)$ if and only if the implication

$$(A \times \{1\}) \cup (B \times \{2\}) \cup (C \times \{3\}) \to \{\leq\} \tag{1}$$

is valid in $\mathbb{K}^{\mathbb{P}}_{\sim}$.

Example

$$\text{SUV} \preccurlyeq_{\{\text{bright},\text{dark}\}} \text{minivan}$$

$$\Updownarrow$$

$$\{\text{SUV}_1, \text{minivan}_2, \text{bright}_3, \text{dark}_3\} \to \{\leq\}$$

Ceteris paribus preferences as implications

Translation of ceteris paribus preferences

A ceteris paribus preference $A \preccurlyeq_C B$ is valid in a preference context $\mathbb{P} = (G, M, I, \leq)$ if and only if the implication

$$(A \times \{1\}) \cup (B \times \{2\}) \cup (C \times \{3\}) \to \{\leq\} \tag{1}$$

is valid in $\mathbb{K}^{\mathbb{P}}_{\sim}$.

Proposition

The set

$$\{A \preccurlyeq_C B \mid (A \times \{1\}) \cup (B \times \{2\}) \cup (C \times \{3\}) \text{ is minimal}$$
$$\text{w.r.t. } \mathbb{K}^{\mathbb{P}}_{\sim} \models (1) \text{ and } A \cap B = A \cap C = B \cap C\}$$

is sound and complete for the preference context $\mathbb{P}$.

Ceteris Paribus Consequence($A \preccurlyeq_C B, \Pi$)

Input: A ceteris paribus preference $A \preccurlyeq_C B$ and a set Π of ceteris paribus preferences (over a universal set M).

Output: **true**, if $\Pi \models A \preccurlyeq_C B$; **false**, otherwise.

$\mathcal{S} := [A \cup (B \cap C) \preccurlyeq_{C \cup (A \cap B)} B \cup (A \cap C)]$ {stack}

repeat

  $D \preccurlyeq_F E := \mathrm{pop}(\mathcal{S})$

  if $A \subseteq D, B \subseteq E$, and $C \subseteq F$ for no $A \preccurlyeq_C B$ in Π **then**

    $X := M \setminus (D \cup E \cup F)$

    if $X = \varnothing$ **then**

      return false

    choose $m \in X$

    push($D \cup \{m\} \preccurlyeq_F E, \mathcal{S}$)

    push($D \preccurlyeq_F E \cup \{m\}, \mathcal{S}$)

    push($D \preccurlyeq_{F \cup \{m\}} E, \mathcal{S}$}

until empty($\mathcal{S}$)

return true

Checking semantic consequence

- The algorithm is exponential in $|M|$.
- The theory implied by ceteris paribus preferences is generated by
  - Horn formulas into which we translate preferences;
  - $\neg m_i \vee \neg m_j \vee m_k$ for each $m \in M$ and $i \neq j \neq k \in \{1, 2, 3\}$;
  - $m_1 \vee m_2 \vee m_3$ for each $m \in M$.
- However, the algorithm is linear in $|\Pi|$.

Universal preferences as implications

- The same, but only in two parts:

	m_1	s_1	r_1	w_1	b_1	d_1	$\leq$	m_2	s_2	r_2	w_2	b_2	d_2
c_1, c_1	×			×		×	×	×			×		×
c_1, c_2	×			×		×			×		×		×
c_1, c_3	×			×		×		×			×	×	
c_1, c_4	×			×		×			×	×			×
c_1, c_5	×			×		×	×	×		×			×
c_2, c_1		×		×		×	×	×			×		×
...													

- The set

$$\{A \preccurlyeq_\forall B \mid (A \times \{1\}) \cup (B \times \{2\}) \text{ is minimal}$$
$$\text{w.r.t. } \mathbb{K}_\forall^\mathbb{P} \models (A \times \{1\}) \cup (B \times \{2\}) \to \{\leq\}\}$$

is a minimal complete set of universal preferences valid in $\mathbb{P}$.

- This reduces the problem of finding universal preferences to the problem of finding minimal premises for $\leq$.

Existential preferences as implications

Existential translation of $\mathbb{P}$

$\mathbb{K}_{\exists}^{\mathbb{P}} = (G, \mathfrak{P}(M), I_{\exists})$

$$gI_{\exists}A \quad \Longleftrightarrow \quad g^{\leq} \cap A' \neq \varnothing.$$

	∅	{d}	{b}	{b, d}	{w}	{w, d}	{w, b}	{w, b, d}	{r}	{r, d}	{r, b}	{r, b, d}	{r, w}	{r, w, d}	{r, w, b}	...
c_1	×	×			×	×			×	×						
c_2	×	×			×	×			×	×						
c_3	×	×	×		×	×	×		×	×						
c_4	×	×	×		×	×	×		×	×						
c_5	×	×							×	×						

$$\{A \preccurlyeq_{\exists} B \mid A \neq B, \qquad A \text{ is minimal and } B \text{ is maximal w.r.t. } \mathbb{K}_{\exists}^{\mathbb{P}} \models \{A\} \rightarrow \{B\}\}$$

is a sound and complete subset of existential preferences of $\mathbb{P}$.

Horn bias

The Horn bias induced by a preference context $\mathbb{P} = (G, M, I, \leq)$ is the set of implications that hold in (G, M, I).

Horn-biased preferences

If	$\mathcal{H}$	is the Horn bias induced by $\mathbb{P} = (G, M, I, \leq)$,
	Π	is the set of all preferences that hold in $\mathbb{P}$,
	π	$\in \Pi$ is a preference,
	Π_1	$\subseteq \Pi \setminus \{\pi\}$, such that $\Pi_1 \not\models \pi$ and $\mathcal{H} \cup \Pi_1 \models \pi$,
then	π	is Horn-biased in $\mathbb{P}$.

bright $\preccurlyeq_\forall$ minivan is Horn-biased in $\mathbb{P}$

$$\left\{ \text{minivan, white, bright} \preccurlyeq_\forall \text{minivan} \right\} \quad \not\models \quad \text{bright} \preccurlyeq_\forall \text{minivan}$$

but

$$\left\{ \begin{array}{c} \text{bright} \to \text{minivan, white} \\ \text{minivan, white, bright} \preccurlyeq_\forall \text{minivan} \end{array} \right\} \quad \models \quad \text{bright} \preccurlyeq_\forall \text{minivan}$$

"Unbiased" universal preferences

Proposition

A universal preference $A \preccurlyeq_\forall B$ is Horn-biased if and only if

$$A \neq A'' \qquad \text{or} \qquad B \neq B''.$$

Thus, unbiased preferences are preferences over formal concepts.

"Unbiased" universal preferences

Proposition

A universal preference $A \preccurlyeq_\forall B$ is Horn-biased if and only if

$$A \neq A'' \qquad \text{or} \qquad B \neq B''.$$

Thus, unbiased preferences are preferences over formal concepts.

Computation

Unbiased preferences correspond to hypotheses of $\mathbb{K}_\forall^\mathbb{P}$ for $\leq$, and there are quite a few algorithms in FCA to compute minimal hypotheses.

Minimal hypotheses are the intents of the most general concepts covering only positive examples of the target attribute.

“Unbiased” universal preferences

Proposition

A universal preference $A \preccurlyeq_\forall B$ is Horn-biased if and only if

$$A \neq A'' \qquad \text{or} \qquad B \neq B''.$$

Thus, unbiased preferences are preferences over formal concepts.

Computation

Unbiased preferences correspond to hypotheses of $\mathbb{K}_\forall^\mathbb{P}$ for $\leq$, and there are quite a few algorithms in FCA to compute minimal hypotheses.

A similar approach to ceteris paribus preferences makes it possible to remove a stronger bias.

Universal preferences and implications

A hybrid inference system

How dow we compute all preferences from the Horn bias and "unbiased" preferences?

1. Armstrong rules for implications
   - as for functional dependencies in database theory
2. The rule for universal preferences
3.

$$\frac{X \to \bot}{\emptyset \preccurlyeq_\forall X, \quad X \preccurlyeq_\forall \emptyset}$$

$$\frac{X \to Y, \quad X \cup Y \preccurlyeq_\forall Z}{X \preccurlyeq_\forall Z} \qquad \frac{X \to Y, \quad Z \preccurlyeq_\forall X \cup Y}{Z \preccurlyeq_\forall X}$$

"Unbiased" existential preferences

Proposition

An existential preference $A \preccurlyeq_{\exists} B$ is Horn-biased if and only if

$$A \neq A'' \qquad \text{or} \qquad B \neq B''.$$

Conceptual existential translation of $\mathbb{P}$

$\mathbb{C}_{\exists}^{\mathbb{P}} = (G, \underline{\mathfrak{B}}(G, M, I), I_{\exists})$ concepts as new attributes

$$gI_{\exists}(A, B) \iff g^{\leq} \cap A \neq \varnothing.$$

"Unbiased" existential preferences

Proposition

An existential preference $A \preccurlyeq_\exists B$ is Horn-biased if and only if

$$A \neq A'' \qquad \text{or} \qquad B \neq B''.$$

Conceptual existential translation of $\mathbb{P}$

$\mathbb{C}^{\mathbb{P}}_\exists = (G, \underline{\mathfrak{B}}(G, M, I), I_\exists)$ concepts as new attributes

$$gI_\exists(A, B) \quad \Longleftrightarrow \quad g^{\leq} \cap A \neq \varnothing.$$

$A \preccurlyeq_\exists B$ is valid in a preference context $\mathbb{P}$ if and only if

$$\{(A', A'')\} \to \{(B', B'')\}$$

is valid in $\mathbb{C}^{\mathbb{P}}_\exists$.

Existential preferences as implications between concepts

A complete set of unbiased preferences relative to implications

$$\{A \preccurlyeq_{\exists} B \mid \mathbb{C}_{\exists}^{\mathbb{P}} \models \{(A', A)\} \to \{(B', B)\} \text{ and } B \not\subseteq A\}$$

Existential preferences and implications: inference

1. Armstrong rules (for implications)
2. The rules for existential preferences
3. Additional rule:

$$\frac{A \to B}{A \preccurlyeq_{\exists} B}$$

Further work

- Compact representations
- Efficient algorithms
- Strict preferences
- "Ordinal ceteris paribus" conditions (via FCA scaling):

  John prefers an academic job to a job in industry (given the salary is at least as good).

- Learning preferences with queries
- Association rules instead of implications
- Preferences under incomplete knowledge
- Experimental evaluation