# Basic Level of Concepts in Formal Concept Analysis

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# Motivation

- Concept lattice usually contains large number of concepts.
- Some of concepts are more important (or natural) than other.
- Goal: Select only important concepts.
- There are several approaches to this problem.
- Our approach is based on a phenomenon well-know in psychology of concepts, which is referred to as the basic level of concepts.

# Our goal

Discover basic level of concepts from point of view FCA.

- Show that basic level of concepts may be beneficial for FCA. (e.g. utilized basic level of concepts in FCA for selecting important concepts)
- Show that FCA provides simple framework for studying phenomenon itself.

# Motivation for our approach

What is this?

# Why dog?

There is a lot of other possibilities:

- Animal
- Mammal
- Canine beast
- Retriever
- Golden Retriever
- Marley

# Basic level phenomenon

- When people categorize (or name) objects, then prefer to use certain concepts.
- Preferred concepts are called the concepts of the basic level.
- Concepts in the basic level are compromise between the most general level and the most specific level.
- Using such concepts cognitively economic. Basic level concept "carve the world well".
- There exist several informal definitions of the basic level.
- We follow one of the first approaches, due to Eleanor Rosch.
- Rosch: Objects of the basic level concepts are similar to each other, objects at the superordinate concepts are significantly less similar, while the objects of the subordinate concepts are only slightly more similar.

# An Approach to Basic Level in FCA

Formal concept $\langle A, B \rangle$ belongs to the basic level if it satisfies following properties:

(BL1) $\langle A, B \rangle$ has a high cohesion.

(BL2) $\langle A, B \rangle$ has a significantly larger cohesion than its upper neighbours.

(BL3) $\langle A, B \rangle$ has a slightly smaller cohesion than its lower neighbours.

Cohesion of formal concept = measure of object similarity.

Upper neighbors of $\langle A, B \rangle$ are the concepts that are more general than $\langle A, B \rangle$ and are directly above $\langle A, B \rangle$ in the hierarchy of concepts.

Lower neighbors of $\langle A, B \rangle$ are the concepts that are more specific than $\langle A, B \rangle$ and are directly below $\langle A, B \rangle$ in the hierarchy of concepts.

**Definition**

$$\mathcal{UN}(c) = \{d \in \mathcal{B}(X, Y, I) \,|\, c < d \text{ and there is no } d' \text{ for which } c < d' < d\},$$

$$\mathcal{LN}(c) = \{d \in \mathcal{B}(X, Y, I) \,|\, c > d \text{ and there is no } d' \text{ for which } c > d' > d\}.$$

# Similarity

Similarity of objects $x_1$ and $x_2$ on $\langle X, Y, I \rangle$ can be view as similarity of their corresponding intents.

$$sim(x_1, x_2) = sim_Y(\{x_1\}^{\uparrow}, \{x_2\}^{\uparrow}). \tag{1}$$

$sim(x_1, x_2)$ denotes the degree (or index) of similarity of objects $x_1$ and $x_2$.

## Definition

For $B_1, B_2 \subseteq Y$

$$sim_{\mathrm{SMC}}(B_1, B_2) = \frac{|B_1 \cap B_2| + |Y - (B_1 \cup B_2)|}{|Y|}, \tag{2}$$

$$sim_{\mathrm{J}}(B_1, B_2) = \frac{|B_1 \cap B_2|}{|B_1 \cup B_2|}. \tag{3}$$

# Cohesion

$coh(c)$ denotes the degree (or index) of cohesion of formal concept $c$.

## Definition

For $\langle A, B\rangle \in \mathcal{B}(X, Y, I)$

$$coh^{\varnothing}(A,B) = \frac{\sum_{\{x_1,x_2\}\subseteq A, x_1\neq x_2} sim(x_1,x_2)}{|A|\cdot(|A|-1)/2}. \quad (4)$$

$$coh^{\mathrm{m}}(A,B) = \min_{x_1,x_2\in A} sim(x_1,x_2), \quad (5)$$

# Basic level degree

- We can compute for every formal concepts $\langle A, B\rangle$ of $\langle X, Y, I\rangle$ degree $BL(A, B)$ to which $\langle A, B\rangle$ is a concept from the basic level.
- Concepts from the basic level need to satisfy conditions (BL1), (BL2), and (BL3), it seems natural to construe $BL(A, B)$ as the degree to which a conjunction of the three propositions, (BL1), (BL2), and (BL3), is true.

$$BL(A, B) = \mathcal{C}(\alpha_1(A, B), \alpha_2(A, B), \alpha_3(A, B)), \tag{6}$$

  where
  - $\alpha_i(A, B)$ is the degree to which condition (BL$i$) is satisfied, $i = 1, 2, 3$,
  - $\mathcal{C}$ is a "conjunctive" aggregation function
- Simple form of $C$

$$\mathcal{C}(\alpha_1, \alpha_2, \alpha_3) = \alpha_1 \otimes \alpha_2 \otimes \alpha_3.$$

- Degrees are numbers in $[0, 1]$, we can use product t-norm $a \otimes b = a \cdot b$.

# Formulas

$$\alpha_1^*(A,B) = coh^*(A,B), \tag{7}$$

$$\alpha_2^{\varnothing *}(A,B) = 1 - \frac{\sum_{c \in \mathcal{UN}(A,B)} coh^*(c)/coh^*(A,B)}{|\mathcal{UN}(A,B)|}, \tag{8}$$

$$\alpha_2^{\mathrm{m}*}(A,B) = 1 - \max_{c \in \mathcal{UN}(A,B)} coh^*(c)/coh^*(A,B), \tag{9}$$

$$\alpha_3^{\varnothing *}(A,B) = \frac{\sum_{c \in \mathcal{LN}(A,B)} coh^*(A,B)/coh^*(c)}{|\mathcal{LN}(A,B)|}, \tag{10}$$

$$\alpha_3^{\mathrm{m}*}(A,B) = \min_{c \in \mathcal{LN}(A,B)} coh^*(A,B)/coh^*(c). \tag{11}$$

- * means $\varnothing$ or $m$
- Values of $\alpha_1(A,B)$, $\alpha_2(A,B)$ and $\alpha_3(A,B)$ (and their variants) may naturally be interpreted as the truth degrees to which the propositions in (BL1), (BL2) and (BL3) are true.

# Meaning of formulas

- If $coh^*(c_1) \leq coh^*(c_2)$, then $\frac{coh^*(c_1)}{coh^*(c_2)} \in [0,1]$ may be interpreted as the truth degree of "$coh^*(c_1)$ is only slightly smaller than $coh^*(c_2)$".
- $1 - \frac{coh^*(c_1)}{coh^*(c_2)} \in [0,1]$ may be interpreted as the truth degree of proposition "$coh^*(c_1)$ is significantly smaller than $coh^*(c_2)$".

**Lemma**

*If $\langle A_1, B_1\rangle \leq \langle A_2, B_2\rangle$ then $coh^{\mathrm{m}}(A_2, B_2) \leq coh^{\mathrm{m}}(A_1, B_1)$.*

However, for $coh^{\varnothing}$ such property no longer holds.

# Example

| | $y_1$ | $y_2$ | $y_3$ | $y_4$ | $y_5$ |
|---|---|---|---|---|---|
| $x_1$ | $\times$ | $\times$ | $\times$ | | $\times$ |
| $x_2$ | $\times$ | $\times$ | | $\times$ | |
| $x_3$ | $\times$ | | $\times$ | $\times$ | $\times$ |

For formal concepts $\langle A_1, B_1\rangle = \{\{x_1, x_2\}, \{y_1, y_2\}\}$ and $\langle A_2, B_2\rangle = \{\{x_1, x_2, x_3\}, \{y_1\}\}$ we have $\langle A_2, B_2\rangle \in \mathcal{UN}(A_1, B_1)$ and yet:

$$\begin{aligned} coh^{\varnothing}(A_1, B_1) &= \frac{sim(x_1, x_2)}{1} = \frac{2}{5} < \frac{\frac{2}{5} + \frac{3}{5} + \frac{2}{5}}{3} = \\ &\frac{sim(x_1, x_2) + sim(x_1, x_3) + sim(x_2, x_3)}{3} = coh^{\varnothing}(A_2, B_2), \end{aligned}$$

for both $sim = sim_{\mathrm{SMC}}$ and $sim = sim_{\mathrm{J}}$.

# Solution of problem

Instead of considering $\mathcal{UN}(A,B)$, (all upper neighbors of $\langle A,B\rangle$), we consider only

$$\mathcal{UN}^{\leq}(A,B) = \{c \in \mathcal{UN}(A,B) \mid coh^{\varnothing}(c) \leq coh^{\varnothing}(A,B)\},$$

i.e. only the upper neighbors with a smaller cohesion.
It seems natural to disregard $\langle A,B\rangle$ as a candidate for a basic level concept if the number of "wrong upper neighbors" is relatively large, i.e. if $\frac{|\mathcal{UN}^{\leq}(A,B)|}{|\mathcal{UN}(A,B)|} < \theta$ for some parameter $\theta$.

Analogous, instead of considering $\mathcal{LN}(A,B)$, we consider only

$$\mathcal{LN}^{\geq}(A,B) = \{c \in \mathcal{LN}(A,B) \mid coh^{\varnothing}(c) \geq coh^{\varnothing}(A,B)\}$$

and similar condition for the number of "wrong lower neighbors" given by $\theta$.

# Modified formulas

$$
\begin{aligned}
\alpha_1^*(A,B) &= coh^*(A,B),\\
\alpha_2^{\varnothing *}(A,B) &= [1 - \frac{\sum_{c\in\mathcal{UN}^{\leq}(A,B)} coh^*(c)/coh^*(A,B)}{|\mathcal{UN}^{\leq}(A,B)|}] \cdot ||\frac{|\mathcal{UN}^{\leq}(A,B)|}{|\mathcal{UN}(A,B)|} \geq \theta||,\\
\alpha_2^{\mathrm{m}*}(A,B) &= [1 - \max_{c\in\mathcal{UN}^{\leq}(A,B)} coh^*(c)/coh^*(A,B)] \cdot ||\frac{|\mathcal{UN}^{\leq}(A,B)|}{|\mathcal{UN}(A,B)|} \geq \theta||,\\
\alpha_3^{\varnothing *}(A,B) &= [\frac{\sum_{c\in\mathcal{LN}^{\geq}(A,B)} coh^*(A,B)/coh^*(c)}{|\mathcal{LN}^{\geq}(A,B)|}] \cdot ||\frac{|\mathcal{LN}^{\geq}(A,B)|}{|\mathcal{LN}(A,B)|} \geq \theta||,\\
\alpha_3^{\mathrm{m}*}(A,B) &= [\min_{c\in\mathcal{LN}^{\geq}(A,B)} coh^*(A,B)/coh^*(c)] \cdot ||\frac{|\mathcal{LN}^{\geq}(A,B)|}{|\mathcal{LN}(A,B)|} \geq \theta||.
\end{aligned}
$$

# Experiments

- We performed several experiments.
- We used relative small datasets.
- Subjectivity factor plays a significant role.
- Datasets describing commonly known objects, for which most people would probably agree with selected basic level concepts.
- For every dataset $\langle X, Y, I \rangle$ we compute the basic level degree of all concepts of the concept lattice $\mathcal{B}(X, Y, I)$.
- $BL_{\mathrm{s}}^{\mathrm{c},\mathrm{a}}(A, B)$: $\mathrm{s}$ is $\mathrm{SMC}$ or $\mathrm{J}$ and indicates whether $sim_{\mathrm{SMC}}$ or $sim_{\mathrm{J}}$ was used; $\mathrm{c}$ is $\varnothing$ or $\mathrm{m}$ and indicates whether $coh^{\varnothing}$ or $coh^{\mathrm{m}}$ was used; $\mathrm{a}$ is $\varnothing$ or $\mathrm{m}$ and indicates whether $\alpha_2^{\varnothing *}$ and $\alpha_3^{\varnothing *}$, or $\alpha_2^{\mathrm{m}*}$ and $\alpha_3^{\mathrm{m}*}$ was used.

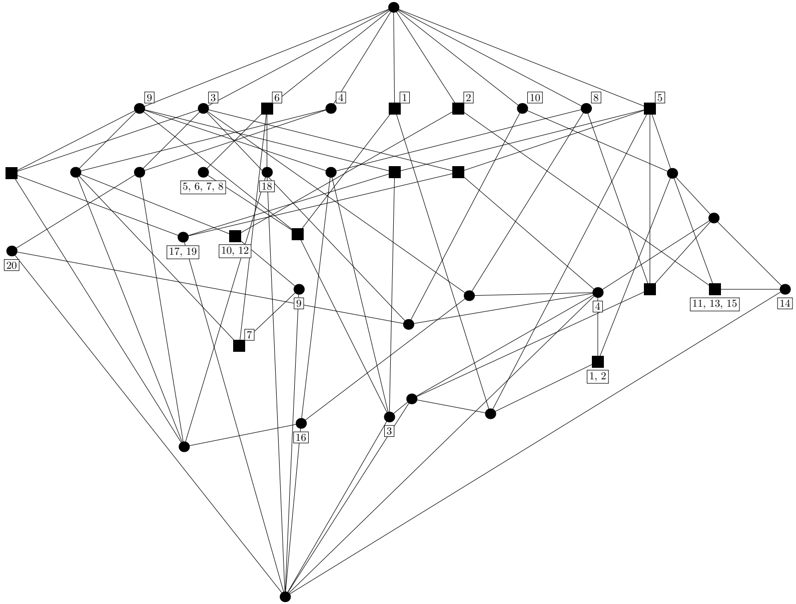
# Experiment (sports)

| | | on land | on ice | in water | collective sport | individual sport | using ball | needs opponent | multiple disciplines | points | time |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Run | 1 | × | | | | × | | | | | × |
| Orienteering | 2 | × | | | | × | | | | | × |
| Gymnastics | 3 | × | | | | × | | | × | × | |
| Triathlon | 4 | × | | × | | × | | | × | | × |
| Football | 5 | × | | | × | | × | × | | × | |
| Inline Hockey | 6 | × | | | × | | × | × | | × | |
| Tennis | 7 | × | | | × | | × | × | | × | |
| Baseball | 8 | × | | | × | | × | × | | × | |
| Ice Hockey | 9 | | × | | × | | | × | | × | |
| Curling | 10 | | × | | × | | | | | × | |
| ⋮ | | | | | | | | | | | |

| | | on land | on ice | in water | collective sport | individual sport | using ball | needs opponent | multiple disciplines | points | time |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| ⋮ | | | | | | | | | | | |
| Cross-country Skiing | 11 | | × | | | × | | | | | × |
| Synchronized Skating | 12 | | × | | × | | | | | × | |
| Alpine Skiing | 13 | | × | | | × | | | | | × |
| Biathlon | 14 | | × | | | × | | | × | | × |
| Speed Skating | 15 | | × | | | × | | | | | × |
| Synchronized Swimming | 16 | | | × | × | | | | × | × | |
| Diving | 17 | | | × | | × | | | | × | |
| Water Polo | 18 | | | × | × | | × | × | | × | |
| Underwater Diving | 19 | | | × | | × | | | | × | |
| Rowing | 20 | | | × | × | | | | | | × |

| intent of concept $\langle A, B\rangle$ | | | | | | | | | | basic level degree of $\langle A, B\rangle$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| on land | on ice | in water | collective sport | individual sport | using ball | needs opponent | multiple disciplines | points | time | $BL^{\emptyset\emptyset}_{\mathrm{SMC}}$ | $BL^{\emptyset\mathrm{m}}_{\mathrm{SMC}}$ | $BL^{\mathrm{m}\emptyset}_{\mathrm{SMC}}$ | $BL^{\mathrm{mm}}_{\mathrm{SMC}}$ | $BL^{\emptyset\emptyset}_{\mathrm{J}}$ | $BL^{\emptyset\mathrm{m}}_{\mathrm{J}}$ | $BL^{\mathrm{m}\emptyset}_{\mathrm{J}}$ | $BL^{\mathrm{mm}}_{\mathrm{J}}$ |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0.33 | 0.33 | 0 | 0 | 0.08 | 0.08 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0.11 | 0.09 | 0 | 0 | 0.05 | 0.04 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0.21 | 0.20 | 0 | 0 | 0.06 | 0.06 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0.15 | 0.12 | 0 | 0 | 0.07 | 0.06 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.31 | 0.29 | 0.12 | 0.11 | 0.08 | 0.07 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0.08 | 0.07 | 0 | 0 | 0.09 | 0.07 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0.10 | 0.07 | 0.21 | 0.14 | 0.10 | 0.10 | 0.10 | 0.09 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0.13 | 0.09 | 0.05 | 0.01 | 0.12 | 0.11 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0.07 | 0.07 | 0 | 0 | 0.08 | 0.08 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.22 | 0.20 | 0 | 0 | 0.07 | 0.06 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0.10 | 0.06 | 0 | 0 | 0.08 | 0.05 |
| 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0.11 | 0.10 | 0.16 | 0.14 | 0.16 | 0.15 | 0.13 | 0.11 |
| 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0.07 | 0.07 | 0.08 | 0.08 | 0.11 | 0.11 | 0.11 | 0.11 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.03 | 0.03 | 0 | 0 | 0.04 | 0.04 |
| | | | | | | | | | ⋮ | | | | | | | | |

# Experiments



# Experiment evaluation

Concepts selected for basic level are:
“winter collective sports”, “individual winter sports”, “land sports evaluated by points”, “winter sports”, “collective sports with opponent”, “individual sports”, “ball games”, “land sports evaluated by time”, “individual water sports”, “land sports”.

Concepts not selected for basic level are, for example:
“individual land sports with multiple disciplines”, “sports evaluated by time”, “collective winter sports with opponent evaluated by points”, “collective sports”, “individual winter sports with multiple disciplines evaluated by time”, “sports performed in water with multiple disciplines”.

# Experiments notes

- We were not checking the results of our method for a given dataset against a psychological experiment.
- Initial study in the presented problem.
- We consider the method promising and giving good results already at this stage.
- An important observation, basic level depends on the dataset and the selected attributes in particular. Typically, a human expert tends to take into account other information (not only the attributes present in the dataset)
- It seems not to matter very much whether $\alpha_2^{\emptyset *}$ and $\alpha_3^{\emptyset *}$, or $\alpha_2^{\mathrm{m}*}$ and $\alpha_3^{\mathrm{m}*}$ is used. On the other hand, it matters significantly whether $coh^{\emptyset}$ or $coh^{\mathrm{m}}$ is used. According to our intuition and the results of this and other experiments we performed, we hypothesize that $coh^{\emptyset}$ is better to use than $coh^{\mathrm{m}}$.
- More detailed study is needed to support this claim.

# Conclusions and Future Research

- We proposed a method that utilizes basic level of concepts to select possibility important concepts from concept lattice and present first results and experience obtained from experiment.
- FCA can be seen as a simple formal framework for study basic level phenomenon.

- Comparison with other techniques to select important formal concepts.
- Utilizing further results of the studies of the basic level in the psychology of concepts.
- Design efficient algorithms to compute basic level concepts.
- Psychological experiments.