Attribute Exploration in a Fuzzy Setting - First Steps

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european social fund in the czech republic
EUROPEAN UNION
MINISTRY OF EDUCATION, YOUTH AND SPORTS
OP Education for Competitiveness
INVESTMENTS IN EDUCATION DEVELOPMENT

Outline

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- Towards a theory of contextual topology (C. Sacarea)
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Definition

$P \subseteq Y$ is called a **pseudo-intent** of (X, Y, I) if

$$P \neq P^{\downarrow\uparrow}$$
$$Q^{\downarrow\uparrow} \subseteq P \text{ holds for every pseudo-intent } Q \subsetneq P.$$

Theorem

The set of implications

$$\mathcal{L} := \{P \to P^{\downarrow\uparrow} \mid P \text{ pseudo-intent}\}$$

is non-redundant and complete.

- Generalised Next Closure Algorithm (Ganter)

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Lemma

Let (X, Y, I) be a context and $P_1, P_2, \ldots, P_n$ be the first n pseudo-intents of (X, Y, I) with respect to the lectic order. If (X, Y, I) is extended by an object g the object intent $g^{\uparrow}$ of which respects the implications $P_i \to P_i^{\downarrow\uparrow}$, $i \in \{1, \ldots, n\}$, then $P_1, P_2, \ldots, P_n$ are also the lectically first n pseudo-intents of the extended context.

Outline

Definitions I

For $A, B \in \mathbf{L}^Y$ the **fuzzy subset-hood** is defined as

$$S(A,B) := \bigwedge_{y \in Y} (A(y) \to B(y)).$$

$A \Rightarrow B$ **holds** in $M \in \mathbf{L}^Y$ with

$$||A \Rightarrow B||_M := S(A,M)^* \to S(B,M)$$

For $\mathcal{M} \subseteq \mathbf{L}^Y$

$$||A \Rightarrow B||_{\mathcal{M}} := \bigwedge_{M \in \mathcal{M}} ||A \Rightarrow B||_M.$$

For a fuzzy context (X,Y,I)

$$||A \Rightarrow B||_{(X,Y,I)} := ||A \Rightarrow B||_{\mathcal{M}},$$

where $\mathcal{M} := \{I_x \mid x \in X\}$.

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Definitions II

T set of fuzzy attribute implications. $M \subseteq \mathbf{L}^Y$ **model** of T if $||A \Rightarrow B||_M = 1$ for each $A \Rightarrow B \in T$. The set of all models of T:

$$Mod(T) := \{M \in \mathbf{L}^Y \mid M \text{ is a model of } T\}.$$

A degree $||A \Rightarrow B||_T \in L$ to which $A \Rightarrow B$ **semantically follows from** T is defined by

$$||A \Rightarrow B||_T := ||A \Rightarrow B||_{Mod(T)}.$$

T is called **complete** (in (X, Y, I)) if

$$||A \Rightarrow B||_T = ||A \Rightarrow B||_{(X,Y,I)}$$

for each $A \Rightarrow B$.
If T is complete and no proper subset of T is complete, then T is called a **non-redundant basis**.

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$\mathcal{P} \subseteq \mathbf{L}^Y$ is called a **system of pseudo-intents** if for each $P \in \mathbf{L}^Y$ we have:

$$P \in \mathcal{P} \Longleftrightarrow P \neq P^{\downarrow\uparrow} \text{ and } ||Q \Rightarrow Q^{\downarrow\uparrow}||_P = 1$$

$$\text{for each } Q \in \mathcal{P} \text{ with } Q \neq P.$$

Theorem (Belohlavek, Vychodil)

$$T := \{P \Rightarrow P^{\downarrow\uparrow} \mid P \in \mathcal{P}\}$$

is complete and non-redundant. If * *is the globalization, then* T *is unique and minimal.*

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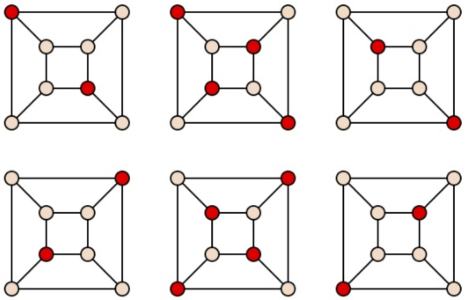
Example

	small (s)	large (l)	far (f)	near (n)
Mercury	1	0	0	1
Venus	1	0	0	1
Earth	1	0	0	1
Mars	1	0	0.5	1
Jupiter	0	1	1	0.5
Saturn	0	1	1	0.5
Uranus	0.5	0.5	1	0
Neptune	0.5	0.5	1	0
Pluto	1	0	1	0

globalization	id 1	id 2
$n \Rightarrow s$	$n \Rightarrow s(.5)$	$n \Rightarrow s(.5)$
$f, n(.5) \Rightarrow l$	$f, n(.5) \Rightarrow l(.5)$	$f, n(.5) \Rightarrow l(.5)$
$l(.5) \Rightarrow f$	$l \Rightarrow f, n(.5)$	$l, f(.5) \Rightarrow f, n(.5)$
$l, f \Rightarrow n(.5)$	$s, l(.5), f(.5) \Rightarrow n(.5)$	$s, l(.5), f(.5) \Rightarrow n(.5)$
$s(.5), n(.5) \Rightarrow s, n$		
$s, l(.5), f \Rightarrow l, n$		

Outline

Maximal independent sets



$$V := \{P \in \mathbf{L}^Y \mid P \neq P^{\downarrow\uparrow}\},$$
$$E := \{(P,Q) \in V \times V \mid P \neq Q \text{ and } ||Q \Rightarrow Q^{\downarrow\uparrow}||_P \neq 1\}, \text{ if } V \neq \varnothing$$

$\mathbf{G} := (V, E \cup E^{-1})$ graph. For $Q \in V$, $\mathcal{P} \subseteq V$ define

$$Pred(Q) := \{P \in V \mid (P,Q) \in E\}$$
$$Pred(\mathcal{P}) := \bigcup_{Q \in \mathcal{P}} Pred(Q).$$

Lemma (Belohlavek, Vychodil)

Let $\varnothing \neq \mathcal{P} \subseteq \mathbf{L}^Y$. If $V - \mathcal{P} = Pred(\mathcal{P})$, then $\mathcal{P}$ is a maximal independent set in $\mathbf{G}$.

Lemma (Belohlavek, Vychodil)

Let $\mathcal{P} \subseteq \mathbf{L}^Y$. $\mathcal{P}$ is a system of pseudo intents if and only if $V - \mathcal{P} = Pred(\mathcal{P})$.

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Theorem (Belohlavek, Vychodil)

$\mathbf{L}$ *residuated lattice with globalization. Then, for each* (X, Y, I) *with finite* Y *there is a unique system of pseudo-intents* $\mathcal{P}$.

- Generalised Next Closure (Belohlavek, Vychodil)
- First n pseudo-intents in $\mathcal{P}$ remain the first n after extending the context.

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The algorithm

$\mathcal{L} := \varnothing$; $A := \varnothing$
if $(A = A^{\downarrow\uparrow})$ **then add** A to $Int(X,Y,I)$
  else Ask expert whether $A \Rightarrow A^{\downarrow\uparrow}$ is valid in (X,Y,I)
    If yes,**add** $A \Rightarrow A^{\downarrow\uparrow}$ to $\mathcal{L}$
    otherwise ask for counterexample x and **add** it to (X,Y,I)
end if
do while $(A \neq Y)$
  for $i = n, \ldots, 1$ **and for** $l = \max L, \ldots, \min L$ **with** $i(l) \notin A$ **do**
    $A := cl_{T^*}(A)$
    if $(A = A^{\downarrow\uparrow})$ **then add** A to $Int(X,Y,I)$
      else
      Ask expert whether $A \Rightarrow A^{\downarrow\uparrow}$ is valid in (X,Y,I)
        If yes, **add** $A \Rightarrow A^{\downarrow\uparrow}$ to $\mathcal{L}$
        otherwise ask for counterexample x and **add** it to (X,Y,I)
    end if
  end for
end do

Example

	small (s)	large (l)	far (f)	near (n)
Earth	1	0	0	1
Mars	1	0	0.5	1
Pluto	1	0	1	0

- $n \Rightarrow s$???
  - Yes!
- $f \Rightarrow s$???
  - No!

Jupiter	0	1	1	0.5

- $f, n(0.5) \Rightarrow l$???
  - Yes!
- $l(0.5) \Rightarrow l, f, n(0.5)$???
  - No!

Uranus	0.5	0.5	1	0

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- Examples.
- Fuzzy attribute exploration with background knowledge.

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Thanx for the attention!

Questions???