

Concepts and Fuzzy Logic: Presentation of a New Book

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Outline

- The Book and Its Story
- Fuzzy Logic in the Psychology of Concepts
- Misunderstandings, Misconceptions and Oversights:
analysis of the most influential paper
- The Book's Content and Message

The Book and its Story



R. Belohlavek, G. J. Klir (Eds.): Concepts and Fuzzy Logic, MIT Press 2011.

Amazon <http://www.amazon.com/Concepts-Fuzzy-Logic-Radim-Belohlavek/dp/0262016478>

\$35 or less

Reviews

Gregory L. Murphy, Department of Psychology, New York

"Thirty years ago, fuzzy set theory and cognitive science had a close relationship, centered on issues in the representation of concepts. There was a falling out, however, and each enterprise has gone its own way since. This volume attempts a rapprochement, through accessible tutorials and critical essays from each field. If the two resume a productive relationship in the future, this may well be through the efforts of the readers of this thoughtful and well-written book."

Gregg C. Oden, Professor Emeritus of Psychology and of Computer Science, U. Iowa

"The editors have assembled a diverse and stellar collection of authors to help them in bridging the interdisciplinary gap between the psychology of concepts and theorizing in fuzzy logic. This is an important undertaking and one that is long overdue, given the as yet largely untapped potential synergy at the interface of these fields."

Lofti A. Zadeh, University of California, Berkeley

"Concepts in Fuzzy Logic stands out as a fundamental contribution to the foundations of fuzzy logic and, in particular, to the basic problem of concept formation. Professors Klir and Belohlavek, the contributors, and MIT Press deserve a loud applause."

Fuzzy Logic in the Psychology of Concepts

Introduction

The psychology of concepts is a research area oriented to the study of the human use of common sense concepts expressed in natural language, each of which represents a class of objects in the real world.

Emergence of fuzzy logic was motivated by the perceived need for representing and dealing with such concepts mathematically.

Yet, fuzzy logic has been completely rejected in the psychology of concepts. WHY?

Psychology of Concepts: An Overview

- Meaning of the term “concept” in the psychology of concepts.
- Classical view of concepts in the psychology of concepts.
- Rejection of the classical view of concepts.
- Main post-classical views of concepts.
- Some of the research issues in the psychology of concepts.
- The Big Book of Concepts by Gregory L. Murphy (MIT Press, 2002).

Meaning of the term “concept” in the Psychology of Concepts

A concept is a mental representation of some category of objects in the world. It is knowledge about the category that is stored in long-term memory and that is used by default (unconsciously) in the processes underlying higher cognitive competencies.

Psychological concepts are closely associated with lexical concepts—the meaning of linguistic terms of natural language.

A taxonomy of concepts is a system by which concept categories in some domain are ordered by set inclusion.

The psychology of concepts plays a central role in cognitive psychology.

Classical view of concepts in the psychology of concepts

Concepts correspond to categories (sets) of objects.

Each category is defined by a set of attributes that are separately necessary and jointly sufficient.

Two facets of each concept are its intension (or intent) and extension (or extent).

Intension of a concept is the set of attributes that define the associated category.

Extension of a concept is the set of objects that conform to the definition of the associated category.

Classical view is based on the notion of concepts in philosophy, which dates back to Aristotle.

This notion is used in FCA (formal concept analysis).

Rejection of the classical view in the psychology of concepts

Classical view had dominated the psychology of concepts until the early 1970s, when it was largely abandoned as a result of groundbreaking experiments by **Eleanor Rosch** (UC-Berkeley).

Rosch showed that **boundaries of concept categories are never sharp, concepts are (graded) fuzzy**. Each category has an intricate structure involving typicality, family resemblance, similarity, and other characteristics.

(She also showed that in each taxonomy of concepts there exists one level of abstraction, called a basic level, that is most fundamental (and useful) in the sense that it reveals maximum information about the category objects with the least cognitive effort.

Concepts at the basic level are acquired before categories at other levels.)

Main post-classical views of concepts in the psychology of concepts

Prototype view: Concepts are represented by prototypes (objects that best satisfy in the psychological sense the associated category) and some similarity measure (in general not symmetric) by which objects are compared with the prototypes.

Exemplar view: Concepts are represented by all examples that a particular person has encountered of that concept. A new object is judged by its similarity to past remembered examples.

Knowledge view: According to this view (also called a theory view), categorization is guided by our knowledge (or theories) about the world.

Some of the research questions of concern in the psychology of concepts

- What are the principles by which humans categorize the world?
- How people acquire and use concepts?
- How people combine concepts?
- How uncertain inference from concept knowledge is made about a new object of the category or about another category?
- How new concepts are formed from those stored in the long-term memory?
- How basic levels in conceptual hierarchies in various domains are determined?
- How concepts are represented in the brain?
- What type of knowledge is stored in concepts?

Attitudes toward fuzzy logic in the psychology of concepts

In the 1970s, some psychologists of concepts were optimistic about the prospective use of fuzzy logic in their area.

In 1981, two highly influential psychologists, **Osherson and Smith** (O&S) published a paper (Cognition 9, pp. 299–318), in which they presented a number of specific arguments against the use of fuzzy logic in the psychology of concepts.

Their conclusion: **fuzzy logic is useless in the psychology of concepts.**

The arguments by O&S are analyzed in detail in

Belohlavek, Klir, Lewis, Way: **Concepts and fuzzy sets: Misunderstandings, misconceptions, and oversights. Int. J. Approximate Reasoning (2009).**

It is also shown in the paper how these erroneous claims were uncritically accepted by the whole community of psychologists of concepts and, as a result, fuzzy logic has been rejected since 1981 as useless in the PoC.

Influence of O&S

Medin (review paper): “More recent studies provide extensive arguments against the use of fuzzy-set theory as an account of complex concepts (see Jones 1982 , Osherson & Smith 1981, 1982, ... Roth & Mervis 1983).”

Roth & Mervis (1982): “Osherson and Smith point out a number of logical inconsistencies that arise when fuzzy set theory is used to form complex concepts from simpler ones.”

... we provide many other examples in the IJAR paper

Types of fallacies by Osherson and Smith

- Assumption that fuzzy set theory is a theory of concepts.
- Arguments regarding logically empty and logically universal concepts.
- Failure to recognize the expressive power of fuzzy logic.
- Arguments regarding set inclusion and universal quantification.
- Errors regarding fuzzy logic in the narrow sense.

Assumption that fuzzy set theory is a theory of concepts

“The standard model of fuzzy concept theory is the original fuzzy set model proposed by Zadeh (1965).” (Cohen & Murphy, 1984)

“Fuzzy set theory cannot specify the relationship among representativeness values in different categories, because the theory does not take the attributes of exemplars directly into account.” (Roth & Mervis, 1983)

Consider, e.g., the statement “Probability theory cannot specify the relationship among personal income and spending, because the theory does not take the inflation rate into account” to see the absurdity of the previous statement.

Arguments regarding logically empty and logically universal concepts

O&S require (on the basis of strong intuitions) that conjunctions of complementary concepts (old and not old) be logically empty and claim that fuzzy logic is not able to satisfy this requirement.

They also require that disjunctions of complementary concepts (old or not old) be logically universal and, again, they claim that fuzzy logic is not able to satisfy this requirement.

The phrase “strong intuitions about concepts” is frequently used by O&S, but they never define, clarify, or explain its meaning.

Answer by Zadeh (Cognition, 1982), probably misunderstood.

Failure to recognize the expressive power of fuzzy logic

Example in O&S regarding the concept of financial wealth and its connection to the concepts of liquidity and investment.

“It is clear that of the three persons whose assets are given in Table 1, a enjoys the greatest liquidity, c the greatest investment and b the greatest wealth. Given domain X and subsets for liquidity (L), investment (I) and wealth (W), we have

$L(a) > L(b)$, $I(c) > I(b)$, $W(b) > W(a)$, and $W(b) > W(c)$.”

	Liquidity	Investment
Person a	105,000	5,000
Person b	100,000	100,000
Person c	5,000	105,000

O&S suggest $W(x) = f[L(x), I(x)]$ and say that the only option is that f is max (fuzzy disjunction, union).

Then, clearly, the two inequalities are violated.

	Liquidity	Investment
Person a	105,000	5,000
Person b	100,000	100,000
Person c	5,000	105,000

O&S do not recognize that f can be a more general operation, either a nonstandard union, such as Łukasiewicz, or an aggregation operation (averaging operation such as arithmetic or geometric average).

Moreover, previous research in psychology itself has shown that “or” is often interpreted by compensatory operators such as averages.

Arguments about set inclusion and universal quantification

Given a universe of discourse X , O&S make an assumption that the truth degree of the proposition

“All As are Bs”

is given by

for each $x \in X$: $A(x) \leq B(x)$,

where A and B denote fuzzy sets that represent the linguistic terms (concepts) A and B , respectively.

Then, they argue: Let X be the set of all animals and let the proposition be

“All grizzly bears are inhabitants of North America”, which is generally considered true. An animal a found far from North America (so $B(a) = 0$) that is considered a grizzly bear with a small degree, say $A(a) = 0.1$, will make the proposition false.

The argument evaporates once the truth degree of the proposition is properly represented as

$$\inf_{x \in X}(A(x) \rightarrow B(x)),$$

where \rightarrow denotes a (truth function of) fuzzy implication.

For the Łukasiewicz implication, for example, we the existence of animal a only implies

$$\begin{aligned} \text{truth}(\text{"All As are Bs"}) &= \inf_{x \in X} \min\{1, 1 - A(x) + B(x)\} \leq \\ &\leq \min\{1, 1 - A(a) + B(a)\} = \min\{1, 1 - 0.1 + 0\} = 0.9. \end{aligned}$$

Errors regarding fuzzy logic in the narrow sense

O&S claim: “The problem is that infinite valued logics generally violate strong intuitions about truth, validity, and consistency.”

They justify this claim by only one example: “Consider Łukasiewicz’s infinite valued logic $L\text{-}\aleph$. The intuitively valid sentence ‘If John is happy, and if John is happy only if business is good, then business is good’ is ruled nontautologous in $L\text{-}\aleph$.”

This is incorrect: Let a and b denote, respectively, the propositions “John is happy” and “business is good”. Then, their sentence is represented by the propositional formula

$$(a \& (a \Rightarrow b)) \Rightarrow b.$$

If $t(a)$ and $t(b)$ denote the truth degrees of a and b , then the truth degree that is assigned to this formula in the Łukasiewicz calculus is

$$\min\{1, 1 \max[0, t(a) + \min(1, 1t(a) + t(b))1] + t(b)\}$$

which is equal to 1 for any truth degrees $t(a)$ and $t(b)$ in $[0, 1]$.

Hence, the sentence is a tautology.

Changing attitudes toward fuzzy logic

- History of the IJAR paper (IP).
- From IP to the book Concepts and Fuzzy Logic.
- Psychologists of concepts contributing to this book:
 - Eleanor H. Rosch (UC Berkeley),
 - James A. Hampton (CU London),
 - Edouard Machery (U Pittsburgh),
 - Jay Verkuilen, Rogier Kievit, and Annemarie Zand Scholten (CUNY New York).
- These and other psychologists of concepts begin to show interest in a research cooperation with members of the fuzzy logic community.

Concepts and Fuzzy Logic

- Chapter 1. Introduction (12 pp)
- Chapter 2. Concepts: A Tutorial (32 pp)
- Chapter 3. Fuzzy Logic and Fuzzy Sets: A Tutorial (44 pp)
- Chapter 4. Categories, Concepts, Fuzzy Sets, and Logical Deduction (32 pp)
- Chapter 5. Fallacious Perceptions of Fuzzy Logic in the Psychology of Concepts (28 pp)
- Chapter 6. Representing Concepts by Fuzzy Sets (28 pp)
- Chapter 7. Formal Concept Analysis: Classical and Fuzzy (32 pp)
- Chapter 8. Conceptual Combinations and Fuzzy Logic (24 pp)
- Chapter 9. Concepts and Natural Language (26 pp)
- Chapter 10. Epilog (6 pp)

Opportunities and challenges of cooperative research between the two communities

For many years Dr. Rosenblueth and I had shared the conviction that the most fruitful areas for the growth of the sciences were those which have been neglected as a no-man's land between the various established fields. It is these boundary regions of science which offer the richest opportunities to the qualified investigator. They are at the same time the most refractory to the accepted techniques of mass attack and the division of labor. If the difficulty of a *psychological* problem is mathematical in essence, ten *psychologists* ignorant of mathematics will get precisely as far as one *psychologist* ignorant of mathematics. If a *psychologist*, who knows no mathematics, works with a mathematician who knows no *psychology*, the one will be unable to state his problem in terms that the other can manipulate and the second will be unable to put the answers in any form that the first can understand.

The mathematician need not have the skill to conduct a *psychological* experiment, but he must have the skill to understand one, to criticize one, and to suggest one. The *psychologist* need not be able to prove a certain mathematical theorem, but he must be able to grasp its *psychological* significance and to tell the mathematician for what he should look.

From: Norbert Wiener, *Cybernetics*, John Wiley & MIT Press, 1948.
“physiology” → “psychology”

Some research directions

... identified in the book ...

including

- challenges for psychology of concepts from fuzzy logic
- challenges for fuzzy logic from psychology of concepts

Example

basic level of concepts

utilize the psychological knowledge, formalize and propose criteria for interestingness of formal concepts (if in FCA) or clusters in data

Relevant references

- G. L. Murphy, The Big Book of Concepts. MIT Press, 2002.
- R. Belohlavek and G. J. Klir, Concepts and Fuzzy Logic. MIT Press, 2011.
- D. N. Osherson and E. E. Smith, On the adequacy of prototype theory as a theory of concepts. Cognition, 9, 1981, pp. 35-58.
- L. A. Zadeh, A note on prototype theory and fuzzy sets. Cognition, 12, 1982, pp. 291-297.
- R. Belohlavek, G.J. Klir, H.W. Lewis, E. Way, Concepts and fuzzy sets: misunderstanding, misconceptions, and oversights. IJAR, 51(1), 2009, pp. 23-34.
- Papers by G. Fuhrmann in Synthese: 75, pp. 317-347 & 86, pp. 1-27.
- Papers by G. Fuhrmann in Information Sciences (51, pp. 143-152) and in Fuzzy Sets in Psychology edited by T. Zetenyi (North Holland, 1988, pp. 155-202).